Risk Aversion and the Financial Accelerator

Giacomo Candian\textsuperscript{1} \hspace{1cm} Mikhail Dmitriev\textsuperscript{2}
Boston College \hspace{1cm} Florida State University

This Draft: February 2015

Abstract\textsuperscript{†}

We extend the Bernanke, Gertler and Gilchrist (1999) financial accelerator model with risk-neutral entrepreneurs to entrepreneurs with CRRA preferences. The optimal contract is identical to debt when there is no default, while when there is default entrepreneurs retain a part of their assets. In partial equilibrium, leverage becomes more sensitive to fluctuations in expected returns to capital when entrepreneurs are risk averse. In general equilibrium, this higher sensitivity tends to stabilize business cycle fluctuations. The response of output to “risk shocks” — shocks to the variance of unobserved idiosyncratic productivity — or shocks to the wealth distribution between households and entrepreneurs is 60 to 70 percent smaller when entrepreneurs are risk averse than when they are risk neutral. However, technology and monetary shocks have quantitatively similar effects on key macro variables, although about 20 percent smaller with risk aversion.

**Keywords:** Financial accelerator; financial frictions; risk; optimal contract; agency costs.

**JEL Classification Numbers:** C68, D81,D82, E44, L26.

\textsuperscript{†}We thank Susanto Basu, Diego Comin, Fabio Ghironi, Peter Ireland, Robert King, Fabio Schiantarelli and conference participants at the Green Line Macro Meeting and Midwest Macro Meeting for useful suggestions. All remaining mistakes are our own.

\textsuperscript{1}Email: giacomo.candian@bc.edu. Web: https://www2.bc.edu/giacomo-candian
\textsuperscript{2}Email: mihail.dmitriev@gmail.com. Web: http://www.mikhaildmitriev.org
1 Introduction

With the recent financial crisis, the analysis of the links between the financial sector and the real economy has attracted a lot of attention among macroeconomists. A standard modeling technique that allows for meaningful feedback between these two parts of the economy in a DSGE framework is based on the contributions by Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (BGG, 1999). In BGG, the relationship between lenders and borrowers (entrepreneurs) is characterized by asymmetric information, which introduces a wedge between expected capital returns and the safe rate of return. This wedge, which we will call the capital wedge or excess returns, is countercyclical and amplifies business cycle fluctuations.¹

Building on this framework, a strand of the literature has focused on cyclical variations in cross-sectional uncertainty as a source of business cycle fluctuations. In particular, Christiano, Motto and Rostagno (CMR, 2014) extend the BGG model and, allowing the idiosyncratic uncertainty that characterizes the entrepreneurs' production of capital in the model to be time-varying. According to their findings, these fluctuations in uncertainty — which they refer to as “risk shocks” — deliver business-cycle-like comovements and explain 60% of the variation in US output over the period 1985-2010.

In both the BGG and CMR model, entrepreneurs are assumed to be risk neutral. While this can be a reasonable assumption for investors based on standard arguments of diversification of risk, it is less applicable to entrepreneurs, who often are single individuals exposed to uninsurable risks. For these entrepreneurs, fluctuations in risk as well as risk itself likely play an important role in their investment decisions. Given the focus of the recent literature on risk shocks as main drivers of the business cycle, it is important to extend the BGG model to entrepreneurs whose economic decisions are directly affected by their risk exposure.

To this end, we generalize the BGG framework to the case of entrepreneurs with constant-relative-risk-aversion (CRRA) preferences, yet maintaining an analytically tractable, log-linear

¹BGG refer to this wedge as the external finance premium because their risk neutral entrepreneurs in equilibrium equate the return to capital to the marginal cost of external finance. In the model we develop below, our risk-averse entrepreneurs will equate the marginal utility from holding one extra unit of capital to the marginal cost of acquiring it using external finance. Therefore, in our model expected excess capital returns need not be equal to the premium for external finance.
framework. Notably, our linearization does not result in certainty equivalence because our steady state, while being deterministic in an aggregate sense, still features non-zero volatility of idiosyncratic productivity. In the steady state of our model, every entrepreneur is still exposed to significant idiosyncratic risk, which has a first-order effect. The magnitude of this effect will depend on the risk preferences of the entrepreneurs and on other characteristics of the model.

Our main results are as follows. First, risk-averse borrowers choose a lower leverage in steady state than their risk-neutral counterparts, *ceteris paribus*. This is intuitive since risk averse agents try to reduce the volatility of their returns, which in the model is achieved by cutting leverage. Second, in partial equilibrium, when entrepreneurs are risk averse, leverage becomes more sensitive to fluctuations in excess returns to capital and to shocks to the variance of unobserved idiosyncratic productivity — so called “risk shocks”. This finding is consistent with the results of Chen *et al.* (2010), who study investment and financing decisions for entrepreneurial firms in a dynamic capital structure model with incomplete markets. The higher sensitivity of leverage to excess returns has important general equilibrium implications and tends to stabilize business cycle fluctuations. Indeed, we find that the response of output to risk shocks and wealth shocks is 60 to 70 percent smaller when entrepreneurs are risk-averse than when they are risk-neutral. Finally, the responses of key macro variables to technology and monetary shocks are more similar for risk-averse and risk-neutral borrowers, although about 20 percent smaller in the former case.\(^2\)

In our framework, as well as in BGG, a risk shock increases defaults and the cost of borrowing, inducing entrepreneurs to borrow less and to reduce their purchases of capital goods. In general equilibrium, lower demand for capital depresses the price of capital, triggering two additional effects. First, it generates the BGG financial accelerator, whereby a lower capital price reduces net worth, which lowers investment demand leading to further rounds of decrease in the price of capital, net worth and demand for capital. Second, a low price of capital increases the expected returns to capital because the price is expected to revert back up to steady state.

\(^2\)We find that the response of key macro variables to government spending shocks is very similar for risk-averse and risk-neutral borrowers, although about 15% smaller in the risk-averse case. We do not report these results in the simulations.
Higher expected returns tend to increase borrowing and investment demand. Since the leverage chosen by risk-averse entrepreneurs is more sensitive to expected returns to capital, this second effect tends to increase investment more when entrepreneurs are risk averse relative to when they are risk neutral. Thus, even though the risk shock causes borrowing to decrease in partial equilibrium, higher future returns to capital almost entirely offset the fall in general equilibrium when entrepreneurs are risk averse. With almost no change in credit, demand for capital does not fall as much. As a result, investment and output decline much more moderately when entrepreneurs are risk averse compared to when they are risk neutral. The effect of expected returns to capital on borrowing also explains the muted effects of wealth shocks for risk-averse entrepreneurs, as we discuss below. Instead, the response of endogenous variables to technology and monetary shocks differs less between the risk-averse and the risk-neutral case, since the credit channel described above is not the primary channel of transmission for these shocks. However, as before, the credit channel still delivers more amplification for risk-neutral vis-a-vis risk-averse borrowers.

On the methodological side, we are the first to our knowledge to incorporate risk aversion in a model of idiosyncratic, uninsurable risk such as BGG, while keeping the analytical tractability of a log-linear framework. Modeling costly state verification problems with risk-averse borrowers has several difficulties which we need to address. To begin with, the optimal contract is no longer a debt contract, as for the case of risk neutrality (Townsend, 1979). Under a standard debt contract, in case of default the lender confiscates all the net worth of the borrower. Such an arrangement is no longer optimal for the risk-averse borrower because it would imply a zero-consumption scenario. We use Tamayo’s (2014) and Gale and Hellwig’s (1985) results in the costly state verification literature who show that, in a static, partial-equilibrium setting, risk-averse entrepreneurs would offer a different optimal contract to the lender. This contract ensures that the borrower retains some of his net worth even in the case of default. We extend Tamayo’s (2014) financial contract to a general equilibrium framework that features optimal history-independent loans with predetermined returns for lenders.3

3Precisely, we derive the optimal one-period contract with deterministic monitoring. For CSV in partial equilibrium, the literature has also focused on dynamic contracts with deterministic monitoring (Wang, 2005), dynamic contracts with stochastic monitoring (Monnett and Quintin, 2005) and self-enforcing stochastic
The second difficulty lies in the aggregation of individual histories in the presence of uninsurable idiosyncratic risk and non-linear preferences, whose combination implies that every entrepreneur chooses a different leverage. This form of heterogeneity normally requires giving up the traditional frameworks with a limited number of agents in favor of a more computational approach, e.g. Krussel and Smith (1998). We instead allow entrepreneurs to be risk-averse and make two assumptions that lead to identical leverage choices for potentially different entrepreneurs. Specifically, we allow only newborn entrepreneurs to work, so that labor income does not affect the financial decision of entrepreneurs. Moreover, we assume that all net worth is reinvested in every period and entrepreneurs consume only in the case of death, which occurs with an exogenous probability. These two assumptions keep the aggregation of individual histories simple and ensure, as in BGG, that only aggregate net worth matters for the economy dynamics.

Our results contribute to the literature of costly state verification in DSGE models where frictions arise because of information asymmetries. The CSV framework brings into the business cycle picture the possibility of endogenous defaults, endogenous spreads and cross-sectional variation among borrowers, therefore naturally accommodating questions regarding risk.\textsuperscript{4} Recent applications include Chugh (2013), who studies risk shocks in a model with costly state verification and finds that cross-sectional firm-level evidence provides little empirical support for the presence of large risk shocks. On the other hand, Ferreira (2014) uses a DSGE model together with a reduced form VAR to empirically identify the risk shocks and finds that these shocks explain about 40\% of the decrease in economic activity during the Great Recession, lending further support to the findings of CMR. Dmitriev and Hoddebagh (2013) study risk shocks in a BGG model with optimal state contingent contracts and find that they have little effects. In terms of endogenous spreads, De Graeve (2008) estimates a BGG model using solely non-financial data and finds that the model-implied risk spread is highly correlated with lower monitoring (Cole, 2013).

grade corporate bond spreads. More recently, Martinez-Garcia (2014) finds that the BGG model is producing too countercyclical and large spread between Baa corporate bond yield and the 20-year Treasury bill rate since the Great Moderation. As we show below, the presence of risk-averse entrepreneurs decreases the volatility of excess returns to capital, suggesting that our model generates spreads dynamics more in line with the data.

The paper proceeds as follows. Section 2 derives the static optimal contract in partial equilibrium. Section 3 introduces aggregate risk and dynamics. Section 4 incorporates the resulting contract into the general equilibrium framework. Section 5 contains our quantitative analysis and results. Section 6 concludes.

2 Static Optimal Contract in Partial Equilibrium

In this section we study the optimal contract between a risk-averse borrower (the entrepreneur) and a risk-neutral lender. In the financial frictions literature popularized by BGG, borrowers are assumed to be risk neutral hence indifferent to aggregate or idiosyncratic risk. In the present context instead, the borrower is a risk-averse agent who is subject to uninsurable risk. Lenders are risk-neutral with respect to the idiosyncratic (i.e. entrepreneur-specific) risk because, as will be true in the general equilibrium model developed below, they can diversify their lending activity across a large number of projects.

The static contract between the lender and borrower follows the traditional CSV framework and resembles the optimal contract developed by Tamayo (2014). Entrepreneurs invest in a risky asset (capital) in the amount of $QK$, where $K$ denotes the quantity of capital purchased and $Q$ its relative price. The return on the investment is $QKR^k\omega$, where $R^k$ indicates aggregate returns to capital and $\log(\omega) \sim \mathcal{N}(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$ the idiosyncratic return component that is specific to the entrepreneur with pdf $\phi(\omega)$. $\omega$ is independently distributed across entrepreneurs. We assume that the lender cannot observe the realization of the idiosyncratic shock to the entrepreneurs unless he pays monitoring costs $\mu$ which are in fixed percentage of total assets. In each state of the world $\omega \in \Omega$, the risk-averse entrepreneur chooses to report $s(\omega)$ and the report is verified in the verification set $\Omega^V \subset \Omega$. Following the literature, we assume that reports are always truthful so that $s(\omega) = \omega$ for all $\omega \in \Omega$, which implies that the repayment function depends
only on $\omega$.\footnote{See Tamayo (2014) for details.}

**Definition 1** A contract under CSV is an amount of borrowed funds $B$, a repayment function $R(\omega)$ in the state of nature $\omega$ and a verification set $\Omega^V$, where the lender chooses to verify the state of the world.

The static problem in the presence of only idiosyncratic risk $\omega$ can be formulated as

$$
\max_{K,R} \int_0^\infty [QKR^k(\omega - R(\omega))]^{1-\rho}\phi(\omega)d\omega
$$

(1)

$$
BR \leq QKR^k \int_0^\infty R(\omega)\phi(\omega)d\omega - \mu QKR^k \int_{\omega \in \Omega^V} \omega \phi(\omega)d\omega
$$

(2)

$$
QK = B + N
$$

(3)

$$
0 \leq R(\omega) \leq \omega \quad \forall \omega
$$

(4)

The first equation is the expected utility of the entrepreneur from the investment return. The second equation is a participation constraint for the lender; it says that he should be paid on average the gross safe rate of return, $R$. The third equation just says that the entrepreneur uses the loan ($B$) and his own net worth ($N$) for acquiring capital. The final inequality constraint states that repayments should be non-negative and cannot exceed the total value of assets. The following Proposition is a special case of Tamayo’s (2014) Theorem 1 case iii).

**Proposition 1** Under the optimal contract that solves the problem (1) subject to (2), (3), (4), the repayment function $R(\omega)$ can be written as that

- $\exists \bar{\omega}$ and $\omega$, such that

$$
R(\omega) =
\begin{cases}
0 & \text{if } \omega < \bar{\omega} \\
\omega - \bar{\omega} & \text{if } \bar{\omega} \leq \omega \leq \bar{\omega} \\
R & \text{if } \omega > \bar{\omega}, \text{ where } \bar{\omega} \geq R \geq \bar{\omega} - \omega \\
\end{cases}
$$

$\Omega^V = [0, \bar{\omega})$
The optimal contract is illustrated in Figure 1. When the lender monitors the borrower \((\omega \leq \bar{\omega})\), he does not seize all assets. If the borrower’s returns are very small \((\omega < \omega)\), the lender receives no repayment; if the borrower is a little more successful \((\omega < \omega < \bar{\omega})\), he keeps a fixed amount \(\omega\) of resources, while the lender seizes the rest. As in Townsend’s debt contract, when the borrower is not monitored, the lender receives a flat payoff. The structure of the optimal contract in the defaulting region is the result of the borrower’s attempt to smooth his return across different states of the world.\(^6\) Therefore, optimal risk sharing requires that the borrower is initially prioritized in the repayment. At the same time the lender is indifferent to the structure of the repayment function, as long as his net payment covers the opportunity cost of his funds on average.

Figure 1: Optimal contract with risk-averse entrepreneurs

Corollary 1 When \(\rho \to 0\), \(\omega \to 0\), \(\bar{R} \to \bar{\omega}\), so that the optimal contract replicates the original BGG contract.

\(^6\)Effectively, in the region \(\omega \in (\omega, \bar{\omega})\) the borrower always receives \(\omega\).
Corollary 1 states that when the borrower becomes risk-neutral the optimal contract converges to the debt contract of BGG. In this case the repayment function is completely characterized by \( \bar{\omega} \), as \( \bar{R} \) becomes equal to \( \bar{\omega} \) and \( \omega \) goes to zero. In other words, the debt contract of BGG is a special case of the richer risk-sharing agreement described in Proposition 1.

An interesting implication of Proposition 1 is that, notwithstanding the complexity of the problem under risk-aversion, the repayment function \( R(\omega) \) is completely characterized by the thresholds \((\omega, \bar{\omega})\) and by the non-default repayment \( \bar{R} \). This allows us to reformulate the contracting problem as follows:

\[
L = \max_{\omega, \bar{\omega}, \bar{R}, \kappa, \lambda} \left( \frac{(\kappa R^k)^{1-\rho} g(\bar{\omega}, \omega, \bar{R})}{1-\rho} + \lambda \left( \kappa R^k h(\bar{\omega}, \omega, \bar{R}) - (\kappa - 1) \bar{R} \right) \right)
\]

where \( \kappa \equiv \frac{QK}{X} \), \( g(\bar{\omega}, \omega, \bar{R}) \) and \( h(\bar{\omega}, \omega, \bar{R}) \) are correspondingly:

\[
g(\bar{\omega}, \omega, \bar{R}) = \int_0^{\bar{\omega}} \omega^{1-\rho} \phi(\omega) d\omega + \bar{\omega}^{1-\rho} \int_{\bar{\omega}}^{\infty} \phi(\omega) d\omega + \int_{\bar{\omega}}^{\infty} (\omega - \bar{R})^{1-\rho} \phi(\omega) d\omega \tag{5}
\]

\[
h(\bar{\omega}, \omega, \bar{R}) = (1-\mu) \int_{\omega}^{\bar{\omega}} \omega \phi(\omega) d\omega - \omega \int_{\omega}^{\bar{\omega}} \phi(\omega) d\omega + \bar{R} \int_{\omega}^{\infty} \phi(\omega) d\omega - \mu \int_{0}^{\omega} \omega \phi(\omega) d\omega \tag{6}
\]

The optimal \( \kappa, \bar{\omega}, \omega, \bar{R} \) are only functions of exogenous variables \( R^k, R \) and parameters \( \sigma_\omega, \mu \).

The first-order conditions for this problem are reported in the Appendix.

Figure 2 shows the relationship between the (annualized) discounted returns to capital \((R^k/R)\) and leverage \( \kappa \). The relationship is positive as higher returns to capital lower expected defaults, thereby reducing agency costs and allowing entrepreneurs to borrow more. From the Figure we also see that for any given excess return to capital, as risk-aversion increases, leverage decreases. This is what we should expect as, when risk aversion rises, entrepreneurs will try to reduce the volatility of their returns by cutting leverage. In other words, a precautionary motive arises that reduces the equilibrium leverage.
3 Dynamic Optimal Contract in Partial Equilibrium With Aggregate Risk

In this section we extend the contract to a dynamic setting where entrepreneurs maximize their expected consumption path and returns to capital are subject to aggregate risk. For the moment, aggregate returns to capital and the risk-free rate are still exogenous. We largely use notation from Dmitriev and Hoddenbagh (2013).

At time $t$, the entrepreneur $j$ purchases capital $K_t(j)$ at a unit price of $Q_t$, which he will rent to wholesale goods producers in the next period. The entrepreneur uses his net worth $N_t(j)$ and a loan $B_t(j)$ from the representative lender to purchase capital:

$$Q_tK_t(j) = N_t(j) + B_t(j).$$

In period $t + 1$, entrepreneur $j$ is hit with an idiosyncratic shock $\omega_{t+1}(j)$ and an aggregate
shock $R_{t+1}^k$, so that he is able to deliver $Q_t K_t(j) R_{t+1}^k \omega_{t+1}(j)$ units of assets. The idiosyncratic shock $\omega_{t+1}(j)$ is a log-normal random variable with distribution $\log(\omega_{t+1}(j)) \sim \mathcal{N}(-\frac{1}{2} \sigma_{\omega,t}^2, \sigma_{\omega,t}^2)$ so that the mean of $\omega$ is equal to 1.$^7$ The realizations of $\omega$ are independent across entrepreneurs and over time. When the realization of $\omega_{t+1}(j)$ exceeds $\bar{\omega}_{t+1}$ the entrepreneur is able to repay the loan at the contractual rate $Z_{t+1}$. That is,

$$B_t Z_{t+1} = Q_t K_t R_{t+1}^k \bar{R}_{t+1}$$

(8)

Following BGG, we assume that entrepreneurs die with constant probability $1 - \gamma$. It is well known, for instance from the work of Krussell and Smith (1999), that if agents are risk-averse and subject to uninsurable idiosyncratic risk, there is no simple way of aggregating individual histories and one would need to keep track of the wealth distribution of all the entrepreneurs. Consider the case where entrepreneurs receive a wage income in every period. In this case, different entrepreneurs would choose different leverages, depending on their net worth. For example, entrepreneurs with a very low net worth would realize that, even in the case of very low idiosyncratic returns to capital, if they survive to the next period, they would be able to make up for their losses with their wages. Given their low net worth today the variance of their net worth tomorrow is still pretty low even for a high leverage, therefore it will be optimal to choose a high leverage. Consider instead an entrepreneur with a very high net worth today. In case of a low idiosyncratic realization tomorrow, he would lose almost all his wealth and end up consuming only his wage. This entrepreneur will choose a lower leverage than the low-net-worth entrepreneur. The issue of different leverages does not arise in BGG because entrepreneurs are risk-neutral and thus are indifferent to the variance of their future wealth.

To resolve the aggregation problem we assume that entrepreneurs work only in the first period of their lives and that they consume all their net worth only upon the event of death. If entrepreneurs survive they do not consume anything and reinvest all their proceeds. In order to keep aggregate dynamics of net worth the same of BGG, we assume that in the first period entrepreneurs provide $\frac{1}{1-\gamma}$ units of labor, so that total labor income is identical in both models.

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$^7$The timing is meant to capture the fact that the variance of $\omega_{t+1}$ is known at the time of the financial arrangement, $t$. 

11
Entrepreneur $j$’s value function is

$$V^e_t(j) = (1 - \gamma) \sum_{s=1}^{\infty} \gamma^s \mathbb{E}_t \left( \left( C^e_{t+s}(j) \right)^{1-\rho} \right)$$

(9)

where $C^e_{t+s}(j)$ is the entrepreneur $j$’s consumption in case of his death,

$$C^e_t(j) = N_t(j)$$

(10)

defined as wealth accumulated from operating firms. The timeline for entrepreneurs is plotted in Figure 3.

Figure 3: Timeline for Entrepreneurs

<table>
<thead>
<tr>
<th>$t$</th>
<th>Period $t$ shocks are realized</th>
<th>Rent capital $K_{t-1}$ to wholesalers and receive return $R^k_t$</th>
<th>Pay off loan from period $t-1$ ($B_{t-1}Z_t$) or default</th>
<th>Life/death of entrepreneur</th>
<th>Take out new loan $B_t$ with lending rate $Z_{t+1}$</th>
<th>Buy capital $K_{t+1}$ to rent in period $t+1$</th>
<th>Period $t+1$ shocks are realized</th>
</tr>
</thead>
</table>

The dynamic problem can be formulated recursively as follows:

$$\max_{k_t, R_{t+1}, \omega_{t+1}, \bar{\omega}_{t+1}} \mathbb{E}_t \left[ \frac{(k_t R_{k,t+1})^{1-\rho} g(\bar{\omega}_{t+1}, \omega_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) \Psi_{t+1}}{1 - \rho} \right]$$

(11)

$$s.t. \Psi_t = 1 + \gamma \mathbb{E}_t \left[ (k_t R_{k,t+1})^{1-\rho} g(\bar{\omega}_{t+1}, \omega_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) \Psi_{t+1} \right]$$

(12)

$$s.t. \beta k_t R_{k,t+1} h(\bar{\omega}_{t+1}, \omega_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) = (k_t - 1) R_t$$

(13)

As in BGG, $R_t$ is the safe rate known at time $t$. Lenders require to be paid $R_t$ on average, which implies that the contract must specify a triplet $\{\omega_{t+1}, \bar{\omega}_{t+1}, R_{t+1}\}$ contingent on $R^k_{t+1}$.\(^8\)

This assumption about the repayment to the lenders makes entrepreneurs effectively bear the aggregate risk. The following Proposition summarizes the solution to the dynamic contracting

\(^8\)Later in the general equilibrium model $R_t$ will be equal to the inverse of the household’s stochastic factor.
Proposition 2 Solving problem (11)-(13) and log-linearizing the solution gives the following relationship between leverage and the expected discounted return to capital

\[ \hat{\kappa}_t = \nu_p (E_t \hat{R}_{t+1} - R_t) \]  \hspace{1cm} (14)

where \( \nu_p > 0 \). Moreover, when the standard deviation of idiosyncratic productivity varies over time, the relationship becomes

\[ \hat{\kappa}_t = \nu_p (E_t \hat{R}_{t+1} - R_t) + \nu_\sigma \hat{\sigma}_{\omega, t} \]  \hspace{1cm} (15)

with \( \nu_\sigma < 0 \).

Proof Equations (14) and (15) are obtained in the Appendix.

Following our assumptions about entrepreneurial wage and consumption, all entrepreneurs choose the same leverage regardless of their net worth, so that aggregate leverage \( \kappa_t \) will simply be equal to the leverage chosen by each entrepreneur. Moreover, to a first-order approximation the complex financial agreement between borrowers and lenders boils down to the single equation (14) that links leverage to the expected excess return or the capital wedge. Note that equation (14) is identical in form to the one in BGG (equation (4.17) in their paper). The presence of risk-aversion only changes the elasticity of leverage to the excess returns \( \nu_p \) and to the volatility of idiosyncratic productivity \( \nu_\sigma \), if \( \sigma_\omega \) is allowed to change over time. In this sense, our framework fully nests the BGG framework and this is what allows us to compare the two models in a meaningful way.

When borrowers are risk averse \( (\rho > 0) \) the values of the elasticities \( \nu_p \) and \( \nu_\sigma \) will be different from the risk neutral case. For all the calibrations that we considered we have that

\[ \frac{\partial \nu_p}{\partial \rho} > 0 \quad \quad \frac{\partial \nu_\sigma}{\partial \rho} > 0 \]

To understand this result it is useful to think about how \( \rho \) affects steady-state leverage and
marginal monitoring costs. Marginal monitoring costs represent the marginal cost of increasing leverage and, importantly, they are a convex function of leverage itself. Therefore, when leverage is lower, marginal monitoring costs are also lower and less sensitive to leverage. An increase in risk aversion reduces steady state leverage, as explained in Section 2. Lower leverage means that the steady state is in a region where marginal monitoring costs are flatter relative to the risk neutral case. Hence, the response of $\kappa_t$ to a given change in excess returns to capital ($\nu_p$) will be larger when steady state leverage is lower because in that region marginal monitoring costs are less sensitive to changes in $\kappa_t$.

Proposition 2 indicates that, for a given change in prices, leverage is more volatile when entrepreneurs are risk averse. If leverage varies more also in general equilibrium we might expect investment and output to be more volatile, so that risk aversion would constitute an additional channel of amplification of shocks through the financial accelerator. However, in general equilibrium, excess returns to capital adjust endogenously to changes in the economic environment and it might well be that this adjustment acts as a stabilizer rather than as an amplifier of shocks. Hence, we proceed with the analysis by embedding the optimal contract just derived in the BGG general equilibrium framework. This allows us to study the effect of the financial accelerator with risk-averse entrepreneurs when expected discounted returns to capital are determined endogenously.

4 The Model in General Equilibrium

We now embed our partial equilibrium framework in a standard dynamic New Keynesian model, where returns to capital and returns to lenders are determined endogenously. There are six agents in our model: households, entrepreneurs, financial intermediaries, capital producers, wholesalers and retailers. A graphical overview of the model is provided in Figure 4. The dotted lines denote financial flows, while the solid lines denote real flows (goods, labor, and capital).
4.1 Households

The representative household maximizes its utility by choosing the optimal path of consumption, labor and money

$$\max \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[ \frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\},$$

(16)

where $C_t$ is household consumption, and $H_t$ is household labor effort. The budget constraint of the representative household is

$$C_t = W_t H_t - T_t + \Pi_t + R_{t-1} D_t - D_{t+1} + R^n_{t-1} \frac{B_t}{P_t} - \frac{B_{t+1}}{P_{t+1}}$$

(17)

where $W_t$ is the real wage, $T_t$ is lump-sum taxes, $\Pi_t$ is lump-sum profits received from final goods firms owed by the household, $D_t$ are deposits in financial intermediaries (banks) that pay a real non-contingent gross interest rate $R_{t-1}$ and $B_t$ are nominal bonds that pay a gross non-contingent interest rate $R^n_{t-1}$.

Households maximize their utility (16) subject to the budget constraint (17) with respect to consumption, labor, bonds and deposits yielding the following first order conditions:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \right\} R_t,$$

(18)

$$C_t^{-\sigma} = \beta R^n_t \mathbb{E}_t \left\{ \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right\},$$

(19)

$$W_t C_t^{-\sigma} = \chi H_t^n.$$

(20)

We define the gross rate of inflation as $\pi_{t+1} = P_{t+1}/P_t$.

4.2 Retailers

The final consumption good consists of a basket of intermediate retail goods which are aggregated together in a CES fashion by the representative household:

$$C_t = \left( \int_0^1 c_{it}^{\frac{1}{\varepsilon-1}} di \right)^{\frac{\varepsilon-1}{\varepsilon}}.$$

(21)
The demand for retailer $i$’s unique variety is 

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\varepsilon} C_t,$$  \hspace{1cm} (22)

where $p_{it}$ is the price charged by retail firm $i$. The aggregate price index is defined as 

$$P_t = \left(\int_0^1 p_{it}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \hspace{1cm} (23)$$

Retailers costlessly differentiate the wholesale goods and sell them to households at a markup over marginal cost. They have price-setting power and are subject to Calvo (1983) price rigidities. With probability $1 - \theta$ each retailer is able to change its price in a particular period $t$. Retailer $i$ maximizes the following stream of real profits:

$$\max_{p_{it}^*} \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} \left[ \frac{p_{it}^* - P_t^w}{P_{t+s}} \left(\frac{p_{it}^*}{P_{t+s}}\right)^{-\varepsilon} \right] Y_{t+s} \right\},$$  \hspace{1cm} (24)

where $P_t^w$ is the wholesale goods price and $\Lambda_{t,s} \equiv \beta^{U_{G,t+s}}_{UC,t}$ is the household’s (i.e. shareholder’s) stochastic discount factor. The first order condition with respect to the retailer’s price $p_{it}^*$ is 

$$\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} \left[ \left(\frac{p_{it}^*}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} \left(\frac{p_{it}^*}{P_{t+s}}\right)^{-\varepsilon} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_t^w}{P_{t+s}} \right] \right\} = 0. \hspace{1cm} (25)$$

From this condition it is clear that all retailers that are able to reset their prices in period $t$ will choose the same price $p_{it}^* = P_t^* \forall i$. The price level will evolve according to 

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1 - \theta)(P_t^*)^{1-\varepsilon} \right]^\frac{1}{1-\varepsilon}. \hspace{1cm} (26)$$

Dividing the left and right hand side of (26) by the price level gives 

$$1 = \left[\theta \pi_t^{\varepsilon-1} + (1 - \theta)(p_t^*)^{1-\varepsilon} \right]^\frac{1}{1-\varepsilon}, \hspace{1cm} (27)$$
where \( p_t^* = P_t^* / P_t \). Using the same logic, we can normalize (25) and obtain:

\[
p_t^* = \frac{\varepsilon}{\varepsilon - 1} \sum_{s=0}^{\infty} \theta^s\mathbb{E}_{t-1} \left\{ \Lambda_{t,s} \left(1/p_{t+s}\right)^{-\varepsilon} Y_{t+s} p_{t+s}^{-\varepsilon} \right\},
\]

(28)

where \( p_{t+s}^w = \frac{P_{t+s}^w}{P_t} \) and \( p_{t+s} = P_{t+s} / P_t \).

### 4.3 Wholesalers

Wholesale goods are produced by perfectly competitive firms and then sold to monopolistically competitive retailers who costlessly differentiate them. Wholesalers hire labor from households and entrepreneurs in a competitive labor market at real wage \( W_t \) and \( W_e^t \) and rent capital from entrepreneurs at rental rate \( R_r^t \). Note that capital purchased in period \( t \) is used in period \( t + 1 \).

Following BGG, the production function of the representative wholesaler is given by

\[
Y_t = A_t K_t^{\alpha} \left( H_t \right)^{1-\alpha} \left( H_e^t \right)^{1-\alpha} \left( 1-\Omega \right),
\]

(29)

where \( A_t \) denotes aggregate technology, \( K_t \) is capital, \( H_t \) is household labor, \( H_e^t \) is entrepreneurial labor, and \( \Omega \) defines the relative importance of household labor and entrepreneurial labor in the production process. Entrepreneurs inelastically supply one unit of labor, so that the production function simplifies to

\[
Y_t = A_t K_t^{\alpha} H_e^t^{1-\alpha} \Omega.
\]

(30)

One can express the price of the wholesale good in terms of the price of the final good. In this case, the price of the wholesale good will be

\[
\frac{P_t^w}{P_t} = p_t^w = \frac{1}{X_t},
\]

(31)

where \( X_t \) is the variable markup charged by final goods producers. The objective function for wholesalers is then given by

\[
\max_{H_t, H_e^t, K_t, t} \frac{1}{X_t} A_t K_t^{\alpha} \left( H_t \right)^{1-\alpha} \left( H_e^t \right)^{1-\alpha} \left( 1-\Omega \right) - W_t H_t - W_e^t H_e^t - R_r^t K_{t-1}.
\]

(32)
Here wages and the rental price of capital are in real terms. The first order conditions with respect to capital, household labor and entrepreneurial labor are

\[
\frac{1}{X_t^\alpha} \frac{Y_t}{K_{t-1}} = R_t^e, \tag{33}
\]

\[
\frac{\Omega}{X_t}(1 - \alpha) \frac{Y_t}{H_t} = W_t, \tag{34}
\]

\[
\frac{\Omega}{X_t}(1 - \alpha) \frac{Y_t}{H_t^e} = W_t^e. \tag{35}
\]

Given that equilibrium entrepreneurial labor in equilibrium is 1, we have

\[
\frac{\Omega}{X_t}(1 - \alpha) Y_t = W_t^e. \tag{36}
\]

### 4.4 Capital Producers

While entrepreneurs hold capital between periods, perfectly competitive capital producers hold capital within a given period, and use available capital and final goods to produce new capital. Capital production is subject to adjustment costs, according to

\[
K_t = I_t + (1 - \delta) K_{t-1} - \frac{\phi_K}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}, \tag{37}
\]

where \( I_t \) is investment in period \( t \), \( \delta \) is the rate of depreciation and \( \phi_K \) is a parameter that governs the magnitude of the adjustment cost. The capital producer’s objective function is

\[
\max_{I_t} K_t Q_t - I_t, \tag{38}
\]

where \( Q_t \) denotes the price of capital. The first order condition of the capital producer’s optimization problem is

\[
\frac{1}{Q_t} = 1 - \phi_K \left( \frac{I_t}{K_{t-1}} - \delta \right). \tag{39}
\]
4.5 Lenders

One can think of the representative lender in the model as a perfectly competitive bank which
costlessly intermediates between households and borrowers. The role of the lender is to diversify
the household’s funds among various entrepreneurs. The bank takes nominal household deposits
$D_t$ and lends out the nominal amount $B_t$ to entrepreneurs. In equilibrium, deposits will equal
loanable funds ($D_t = B_t$). Households receive a predetermined real rate of return $R_t$ on their
deposits.

4.6 Entrepreneurs

We have already described the entrepreneur’s problem and timing in detail in Section 3. At the
beginning of each period entrepreneurs rent out the capital they bought at the end of the previ-
ous period to perfectly competitive wholesalers. Later wholesalers return to the entrepreneurs
depreciated capital and pay them the rental rate. After that entrepreneurs sell their capital and
settle their position with the banks, either by repaying their loans or by defaulting. Following
the arrangements with the banks, nature decides which entrepreneurs are going to survive, and
which entrepreneurs are going to die and consume all of their net worth. Subsequently, new
entrepreneurs are born with zero net worth and supply inelastically one unit of labor in the
aggregate. Then newborn and surviving old entrepreneurs borrow money from banks and buy
capital from capital producers.

Wholesale firms rent capital at rate $R_{t+1}^k = \frac{\alpha_Y}{x_t K_{t-1}}$ from entrepreneurs. After production
takes place entrepreneurs sell the undepreciated capital back to capital goods producers for the
unit price $Q_{t+1}$. Aggregate returns to capital are then given by

$$R_{t+1}^k = \frac{\frac{\alpha Y_{t+1}}{x_t} + Q_{t+1}(1 - \delta)}{Q_t}.$$  \hspace{1cm} (40)

Consistent with the partial equilibrium specification, entrepreneurs die with probability $1 - \gamma$,
which implies the following dynamics for aggregate net worth:

$$N_{t+1} = \gamma(Q_t K_t R_{t+1}^h - (Q_t K_t - N_t) R_t - \mu Q_t K_t R_{t+1}^h \int_0^{\tilde{\omega}_{t+1}} \omega \phi(\omega) d\omega) + W_{t+1}^e.$$  \hspace{1cm} (41)
The terms inside the brackets reflect the aggregate returns to capital to entrepreneurs net of loan repayments and monitoring costs. Aggregate entrepreneurial consumption is given by

\[ C^e_t = (1 - \gamma)(N^e_t - W^e_t) \]  

(42)

Given that each entrepreneur chooses the same leverage, we can define leverage as the ratio of aggregate capital expenditure to aggregate net worth

\[ \kappa_t = Q_t K_t / N_t. \]  

(43)

4.7 Goods market clearing

The goods market clearing condition is

\[ Y_t = C_t + I_t + G_t + C^e_t + \mu Q_{t-1} K_{t-1} R^k_t \int_0^{\bar{\omega}} \omega \phi(\omega) d\omega \]  

(44)

where the last term reflects aggregate monitoring costs.

4.8 Monetary Policy

As in BGG, we assume that there is a central bank which conducts monetary policy by choosing the nominal interest rate \( R^n_t \) according to the following rule

\[ \log(R^n_t) - \log(R^n) = \rho R^n \left( \log(R^n_{t-1}) - \log(R) \right) + \xi \pi_{t-1} + \epsilon^n R \]  

(45)

where \( \rho R^n \) and \( \xi \) determine the relative importance of the past interest rate and past inflation in the central bank’s interest rate rule. Shocks to the nominal interest rate are given by \( \epsilon^n R \). It should be noted that the interest rule in BGG differs from the conventional Taylor rule, where current inflation rather than past inflation is targeted.
4.9 Shocks

The shocks in the model follow a standard AR(1) process. The AR(1) processes for technology, government spending and idiosyncratic volatility are given by

\[
\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon^A_t, \tag{46}
\]
\[
\log\left(\frac{G_t}{Y_t}\right) = (1 - \rho^G) \log\left(\frac{G_{ss}}{Y_{ss}}\right) + \rho^G \log\left(\frac{G_{t-1}}{Y_{t-1}}\right) + \epsilon^G_t, \tag{47}
\]
\[
\log(\sigma_{\omega,t}) = (1 - \rho^{\sigma_\omega}) \log(\sigma_{\omega,ss}) + \rho^{\sigma_\omega} \log(\sigma_{\omega,t-1}) + \epsilon^\sigma_\omega_t \tag{48}
\]

where \(\epsilon^A\), \(\epsilon^G\) and \(\epsilon^{\sigma_\omega}\) denote exogenous shocks to technology, government spending and idiosyncratic volatility, and \((G_{ss}/Y_{ss})\) and \(\sigma_{\omega,ss}\) denote the steady state values for government spending relative to output and idiosyncratic volatility respectively. Recall that \(\sigma^2_\omega\) is the variance of idiosyncratic productivity, so that \(\sigma_\omega\) is the standard deviation of idiosyncratic productivity. Nominal interest rate shocks are defined by the BGG Rule in (45).

4.10 Equilibrium

The nonlinear model has 26 endogenous variables and 26 equations. The endogenous variables are: \(R, R^n, H, C, \pi, p^*, p^w, \bar{X}, Y, W, W^e, I, Q, K, R^k, N, C^e, k, \omega, \bar{\omega}, \bar{R}, \Psi, \lambda, G, A, \sigma_\omega\), where the new variable \(\lambda\) corresponds to the Lagrange multiplier for the optimality conditions used in the Appendix. The equations defining these endogenous variables are: (18), (19), (20), (27), (28), (30), (31), (34), (36), (37), (39), (40), (41), (42), (43), (44), and financial contract participation (13), discounting condition (12) and optimality conditions (84), (85), (86), (87). The exogenous processes for technology, government spending and idiosyncratic volatility follow (46), (47) and (48) respectively. Nominal interest rate shocks are defined by the Taylor rule in (45).

4.11 Log-linear Model

The log-linear model has 19 equations and 19 variables, because algebraic manipulations with the Calvo model allow to replace (27), (28) and (31) with (52), and drop \(p^*\) and \(p^w\), while simplifying the financial contract allows to replace (12), (13), (84), (85), (86), (87) with (63)
and drop $\bar{\omega}, \tilde{\omega}, \bar{R}, \Psi$. The equations are

$$- \sigma \left( \mathbb{E}_t \hat{C}_{t+1} - \bar{C}_t \right) + \bar{R}_t = 0,$$

$$\hat{R}_t^n = \hat{R}_t + \mathbb{E}_t \hat{\pi}_{t+1},$$

$$\hat{Y}_t - \hat{H}_t - \bar{X}_t - \sigma \bar{C}_t = \eta \hat{H}_t,$$

$$\hat{\pi}_t = - \frac{(1 - \theta)(1 - \theta \beta)}{\theta} \bar{X}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1},$$

$$\hat{Y}_t = \hat{A}_t + \alpha \bar{K}_{t-1} + (1 - \alpha)(1 - \Omega) H_t,$$

$$\hat{K}_t = \delta \hat{I}_t + (1 - \delta) \hat{K}_{t-1},$$

$$\hat{Q}_t = \delta \phi (\hat{I}_t - \hat{K}_{t-1}),$$

$$\hat{R}_{t+1}^n = (1 - \epsilon) (\hat{Y}_{t+1} - \hat{K}_t - \bar{X}_{t+1}) + \epsilon \hat{Q}_{t+1} - \hat{Q}_t,$$

$$Y \hat{Y}_t = C \hat{C}_t + I \hat{I}_t + G \hat{G}_t + C^n \hat{C}_{t}^n + \phi N (\hat{\phi}_t + \hat{N}_{t-1}),$$

$$\hat{\phi}_t = \hat{Q}_{t-1} + \hat{K}_{t-1} - \hat{N}_{t-1} + \nu_p \hat{\sigma}_{\omega,t-1} + \nu_p \left( \mathbb{E}_{t-1} R_{k,t} - \hat{R}_{t-1} \right),$$

$$\hat{N}_t = \gamma (\kappa R_k (\hat{K}_{t-1} + \hat{R}_{k,t}) - \kappa R \hat{K}_{t-1} - (\kappa - 1) R \hat{R}_{t-1} - \phi \hat{\phi}_t) + \frac{W^e}{N} (\hat{W}_i^e) + \frac{N - W^e}{N} \hat{N}_{t-1},$$

$$\hat{\kappa}_t = \hat{K}_t + \hat{Q}_t - \hat{N}_t,$$

$$C^n \hat{C}_{t}^n = (1 - \gamma) (N \hat{N}_t - W^e \hat{W}_i^e),$$

$$W_i^e = \hat{Y}_t - \bar{X}_t,$$

$$\hat{W}_i^e = \hat{Y}_t - \bar{X}_t,$$

$$\tilde{\kappa}_t = \nu_p (\mathbb{E}_t \hat{R}_{t+1}^n - \hat{R}_t) + \nu \tilde{\sigma}_{\omega,t},$$

$$\hat{A} = \rho^{A} \hat{A}_{t-1} + \epsilon_t^{A},$$

$$\hat{R}_t^n = \rho^{R^n} \hat{R}_{t-1}^n + \xi \hat{\pi}_t + \rho^{Y} \hat{Y}_t + \epsilon_t^{R^n},$$

$$\hat{G}_t = \rho^{G} \hat{G}_{t-1} + \epsilon_t^{G},$$

$$\tilde{\sigma}_{\omega,t} = \tilde{\sigma}_{\omega,t-1} + \epsilon_t^{\sigma_{\omega}}$$

5 Quantitative Analysis

In section 3 we discussed the role of risk aversion in determining the elasticities of leverage with respect to the expected discounted returns to capital and to the standard deviation of idiosyncratic productivity. In particular, we have highlighted the fact that in partial equi-
librium leverage becomes more responsive to the latter with higher risk aversion as marginal monitoring costs build up more slowly. While the partial equilibrium analysis suggests higher sensitivity of leverage and hence higher amplification under risk aversion, the general equilibrium effect depends on the endogenous adjustment of prices and returns. In this section we investigate quantitatively the general equilibrium effects of technology shocks, monetary shocks and idiosyncratic volatility shocks for different coefficients of risk aversion.

5.1 Calibration and Benchmarks

Our baseline calibration largely follows BGG. We set the discount factor $\beta = 0.99$, the risk aversion parameter $\sigma = 1$ so that the utility of households is logarithmic in consumption, and the elasticity of labor supply to 3 ($\eta = 1/3$). The share of capital in the Cobb-Douglas production function is $\alpha = 0.35$. Capital adjustment costs are $\phi_k = 10$ to generate an elasticity of the price of capital with respect to the investment capital ratio of 0.25. Quarterly capital depreciation is $\delta = 0.025$. Monitoring costs are $\mu = 0.12$. The death rate of entrepreneurs is $1 - \gamma = 0.0275$, yielding an annualized business failure rate of eleven percent. The weight of household labor relative to entrepreneurial labor in the production function is $\Omega = 0.99$.

For price-setting, we set the Calvo parameter $\theta = 0.75$, so that 25% of firms can reset their prices in each period, meaning the average length of time between price adjustments is four quarters. As our baseline, we follow the BGG monetary policy rule and set the autoregressive parameter on the nominal interest rate to $\rho^{R_n} = 0.9$ and the parameter on past inflation to $\xi = 0.11$. We set the persistence of the shocks to technology at $\rho^{A} = 0.99$, and keep the standard deviation at 1 percent. Following BGG, for monetary shocks we consider a 25 basis point shock (in annualized terms) to the nominal interest rate with persistence $\rho^{R_n} = 0.9$.

For our purposes, the most important part of the calibration regards the volatility to idiosyncratic productivity and the risk-aversion parameter. We want to compare the impulse responses of the model with risk-averse entrepreneurs to those of the benchmark model with risk-neutral ones. Following Christiano, Motto and Rostagno (CMR, 2014), we set the persistence of idiosyncratic volatility at $\rho^{\sigma_\omega} = 0.9706$. As to the standard deviations of idiosyncratic volatility shocks $\sigma_\omega$, we choose two different values for each coefficient of risk-aversion. If we
set $\sigma_\omega$ to be the same for the different coefficients of risk-aversion, the model with the smaller $\rho$ would imply a higher steady-state leverage. It follows that a shock of a given size would have a stronger effect on impact, since similar movements in prices and returns to capital would induce larger fluctuations in net worth when leverage is higher. Thus, when we increase risk-aversion, we decrease the idiosyncratic volatility to numerically align the steady-state leverage and the excess returns to capital in two models.\footnote{We do not report the results for the two models with different risk-aversion and other identical parameters. In the model with higher risk-aversion and lower leverage the effect on the endogenous variables on impact is smaller for all shocks.}

Following BGG, when entrepreneurs are risk-neutral we set $\sigma_\omega$ to 0.28, which implies a steady-state leverage 2.1 and a value of $R^K/R$ of 1.0084, corresponding to an annualized excess return of 3.3 percent. In the case of risk-averse entrepreneurs we set $\rho = 0.5$ and $\sigma_\omega = 0.085$, which generate leverage of 2.1 and $R^K/R$ of 1.0076, corresponding to annualized excess returns of 3 percent. Why this particular coefficient of risk aversion and level of idiosyncratic volatility?

If we look at the literature on cross-sectional volatility of sales growth, Castro, Clementi and Lee (2010) obtain a value for firm-specific volatility of TFP between 0.04 and 0.12. Comin and Mulani (2006), Davis, Haltiwanger, Jarmin and Miranda (2006) and a more recent study by Veirman and Levin (2014) report the volatility for the annual growth of sales to be between 0.24 and 0.3, however that volatility corresponds to a much smaller standard deviation of quarterly idiosyncratic productivity. We simulate our model in the steady state, where aggregate shocks are absent, but idiosyncratic shocks still affect firms and find that $\sigma_\omega = 0.08$ and $\sigma_\omega = 0.1$ imply a value of volatility of annual sales of 0.24 and 0.3, which is the range observed in the data. We settle for a value of $\sigma_\omega$ of 0.085 and subsequently choose a value for $\rho$ that delivers a leverage of two. The results reported in our simulations are robust to the choice of $\rho$ and $\sigma_\omega$ as long as we select these two parameters to match the leverage and the average excess returns observed in the data.

5.2 Leverage, Capital Returns and Amplification

Our calibration implies that the two cases we consider — risk-averse and risk-neutral entrepreneurs — have very similar steady states in terms of leverage and capital returns. The
first two columns of Table 1 show that in the risk-neutral calibration, the steady-state leverage and $R^k$ are 2.1 and 1.0186, respectively. The risk-averse calibration delivers similar values — leverage of 2.1 and $R^k$ equal to 1.0176 — using a higher risk aversion and a lower volatility of idiosyncratic productivity. We do not report the other steady-state variables but they are very similar across the two models.\(^\text{10}\)

Table 1: Steady-state comparison

<table>
<thead>
<tr>
<th></th>
<th>$\kappa$</th>
<th>$R^k$</th>
<th>$\nu_p$</th>
<th>$\nu_\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-neutral case ($\sigma_\omega=0.28$, $\rho=0.0$)</td>
<td>2.098</td>
<td>1.0186</td>
<td>18.74</td>
<td>-0.71</td>
</tr>
<tr>
<td>Risk-averse case ($\sigma_\omega=0.085$, $\rho=0.5$)</td>
<td>2.084</td>
<td>1.0176</td>
<td>125.36</td>
<td>-1.94</td>
</tr>
</tbody>
</table>

\[ \hat{\kappa}_t = \nu_p (\mathbb{E}_t \hat{R}_{t+1}^k - \mathbb{E}_t \hat{R}_t) + \nu_\sigma \hat{\sigma}_{\omega,t} \]

Despite the fact that steady states are similar, entrepreneurial risk-aversion still affects the way in which the economy reacts to shocks. This different sensitivity is captured by the different values of the two elasticities $\nu_p$ and $\nu_\sigma$ in equation (15) for the two calibrations. Table 1 shows that these elasticities are higher in absolute value for the risk-averse case. As we discussed in section 3, an increase in $\rho$ increases both elasticities in absolute value. The decrease in $\sigma_\omega$ further increases $\nu_p$ and decreases $\nu_\sigma$ although most of the change in the elasticities between our two preferred scenarios is really driven by the increase in $\rho$.\(^\text{11}\) Notably, in our risk-averse calibration the elasticity $\nu_p$ grows by about seven times whereas the elasticity $\nu_\sigma$ grows only by about three times relative to our risk-neutral calibration.

How would higher sensitivity of leverage to excess returns and to the volatility of idiosyncratic productivity affect business cycles? In partial equilibrium, for a given change in prices or idiosyncratic volatility, the larger fluctuations in leverage should strengthen amplification. However, in general equilibrium the impact of $\nu_p$ and $\nu_\sigma$ is less obvious because the movement of prices is endogenous and it differs with and without risk-aversion.

\(^\text{10}\)From the model equations one can see that if leverage, capital returns and defaults are identical, then the two steady states will coincide. Although with higher risk aversion defaults are smaller, they are in both cases very small compared to GDP so that in practice the steady states are almost identical.

\(^\text{11}\)Starting from $\sigma_\omega = 0.28$ and $\rho = 0$ and reducing $\sigma_\omega$ to 0.085 only increases $\nu_p$ from 18.74 to 43.17 and decreases $\nu_\sigma$ from -0.71 to -0.89. Therefore, most of the change in the elasticities is due to the change in $\rho$, rather than to the change in $\sigma_\omega$. 

25
To predict the outcome it is helpful to think about the elasticity \( \nu_p \) in two extreme cases: the frictionless case and the risk-neutral case. In a world without financial frictions \( \nu_p \to \infty \). Even the smallest increase in expected capital returns makes entrepreneurs be willing to hold an infinite amount of capital, owing to constant returns, so that in equilibrium returns to capital are equal to the safe rate. At the opposite end of the spectrum, when entrepreneurs are risk-neutral, \( \nu_p \) is small, reflecting the fact that even if capital returns rise, borrowing cannot increase much because marginal borrowing costs increase very quickly with leverage. Here large swings in excess returns are required to generate movements in leverage. Given that \( \nu_p \) in the risk-averse case is larger than in the risk-neutral case, we should expect excess returns in the risk-averse case to still react to shocks (because financial frictions are still present), but more mildly than in the risk-neutral case. With smaller movements in the returns to capital and, therefore, the price of capital, we expect smaller fluctuations in net worth and less volatile business cycles. Our simulations in the following section confirm our intuition.

5.3 Simulations

In this section we simulate our model and study the impulse responses of key macro variables to different shocks, comparing the case of risk aversion and the case of risk neutrality.

5.3.1 Risk and Wealth Shocks

After a risk shock, the probability of a low realization of \( \omega \) increases, thus banks increase the interest rates charged on loans to cover the higher costs. Entrepreneurs respond by borrowing less and by reducing the quantity demanded of capital goods, given the fewer resources available to them. In general equilibrium, the drop in investment demand reduces the price of capital, which has two additional effects. On the one hand, the lower price of capital reduces the net worth of the entrepreneur which further decreases the demand for capital goods through the standard financial accelerator mechanism described in BGG. On the other hand, there is an additional general equilibrium effect which partially offset the fall in credit, as explained in CMR. Precisely, when the price of capital falls, it is expected to revert to steady state in the future. Other things equal, this raises the expected returns to capital, increasing credit received by entrepreneurs and the demand for capital. For this reason the decline in credit is smaller
than the decline in net worth.

Figure 5 shows the impulse responses to a risk shock for risk-neutral and risk-averse entrepreneurs. In both cases, these dynamic responses are consistent with the intuition given above. Credit falls, net worth falls even more, resulting in an increase in leverage. Investment falls as a result of lower demand caused by the lack of entrepreneurial financial resources. The fall in investment is greatly responsible for the drop in output. Even though the responses are qualitatively similar in the two cases, they are very different quantitatively, with the presence of risk-averse entrepreneurs greatly buffering the fall in investment and output. The difference in the dynamic responses is due to the different response of credit across the two cases. In the risk neutral case, the positive effect on borrowing of higher future returns to capital is weak (given the low value of $\nu_p$) and only mildly offsets the negative impact of the risk shock on borrowing. As a result credit falls significantly when entrepreneurs are risk neutral. Instead when entrepreneurs are risk averse, their demand for investment goods is much more sensitive to changes in prospective returns to capital (this is captured by the higher value of $\nu_p$). Thus, even though the risk shock causes credit to decrease in partial equilibrium, higher future returns to capital, almost entirely offset the fall in general equilibrium. With almost no change in credit, demand for capital does not fall as much. As a result investment and output decline much more moderately when entrepreneurs are risk averse.

Figure 6 shows the impulse responses to a wealth shock that transfers in a lump-sum fashion 1% of the initial net worth of entrepreneurs to households. The intuition for these responses is similar to the intuition for the risk shock. A drop in wealth reduces the net worth of entrepreneurs and the analysis of the debt contract suggests that this reduces borrowing. With fewer credit entrepreneurs reduce the demand for capital goods, driving down their price. The change in the price of capital triggers the general equilibrium effects described above. As noted by CMR, after a wealth shock, the positive effect on credit that works through the increase in expected returns to capital is stronger than the financial accelerator effect, resulting in an increase in credit in equilibrium.\footnote{CMR’s shock is an equity shock rather than a wealth shock. In particular they assume a stochastic process for the parameter $\gamma$, the fraction of entrepreneurs who survive. An unexpected fall in $\gamma$ reduces net worth immediately. Their equity shock and our wealth shock are essentially equivalent.} Nevertheless the increase in credit is not sufficient to cover
the fall in net worth, hence the resources available to entrepreneurs fall. For this reason the wealth shock causes a decline in investment and output. Similarly for the case of the risk shock, our analysis suggests that the expected-returns-to-capital effect is even stronger when entrepreneurs are risk averse, because these types of entrepreneurs are more sensitive to changes in returns to capital. As a result, credit rises by more, net worth falls by less and, hence, the investment and output drops are considerably smaller.

5.3.2 Technology and Monetary Shocks

Figure 7 plots the impulse responses of the two models under risk-neutrality and risk-aversion to a technology shock. In both cases the direction of the responses is the same and follows the intuition of BGG. In particular, the productivity shock immediately stimulates the demand for capital leading to an investment boom. The increase in investment raises asset prices, which raises net worth and reduces the capital wedge. The decline in the wedge further stimulates investment and the financial accelerator mechanism arises: an initial increase in investment increases asset prices and net worth, which further stimulates investment. The financial accelerator model also deliver more persistence than standard New Keynesian models because net worth reverts to steady state very slowly, as can be seen from the Figure. As usual for all models with sticky prices, a one percent increase in total factor productivity leads to less than one percent response of GDP for both models, since marginal costs go down, while prices do not adjust completely on impact, and as a result markups in the economy go up.

The responses of output, investment, consumption and other macroeconomic variables is similar across the two scenarios. The output response is almost identical because consumption and investment behave very similarly in the two cases. As we expected, the response of excess returns to capital is much milder in the risk-averse case, about one fifth of the response of the risk-neutral case. Movements in net worth and leverage are somewhat larger in the risk-averse case but the price of capital increases in a very similar fashion across the two scenarios, which leads to similar responses in investment.

One of the appealing feature of general equilibrium models with costly state verification and risk-neutral borrowers is that they amplify monetary shocks and they make the responses of
macro variables more persistent, thanks to the endogenous dynamics in net worth. Figure 8 shows the impulse responses of the two models with varying degrees of risk-aversion with respect to 25 basis shock to the interest rate. The model with higher risk aversion and more precautionary behavior displays responses to monetary shocks that are about twenty percent smaller on impact vis-a-vis the risk-neutral case. As for the technology shock, excess returns to capital go down much less in the case of risk-aversion. Nevertheless, because of the higher sensitivity of leverage to this wedge, the response of leverage and net worth is quantitatively similar - only about 20% smaller in the risk-averse scenario. The price of capital and investment go up to a smaller extent in the risk-aversion case, therefore, we observe a somewhat smaller reaction of output to the same shock. Nevertheless, the responses are similar in the two cases. The endogenous adjustment of excess returns to capital is such that the financial accelerator mechanism is fundamentally robust to the presence of risk-averse entrepreneurs in response to technology and monetary shocks.

6 Conclusion and Future Research

In this paper we extend the BGG framework to allow borrowers to have constant relative risk-aversion preferences instead of risk-neutral. This new framework is tractable as the popular BGG model of financial frictions but allows us to address new questions regarding risk that were not answerable in the original model. We use this new framework to ask the most natural of the questions that is whether traditional macroeconomic shocks as well as recently popularized shocks have different effects when entrepreneurs are risk averse as opposed to risk neutral. We find that the model with risk-averse borrowers compared to the model with risk-neutral borrowers demonstrates similar responses for technology and monetary shocks, but significantly weaker response for shocks to the volatility of idiosyncratic productivity or “risk-shocks” and for wealth shocks. These results relate to the literature that stresses the importance of changes in uncertainty or idiosyncratic risk in explaining salient features of business cycles, such as Christiano, Motto and Rostagno (2014) and Gilchrist, Sim and Zakrajsek (2014). Our simulations suggest that the quantitative importance of financial shocks such as risk shocks or wealth shocks is sensitive to the risk attitude of the entrepreneurs in the model.
For subsequent research our framework can be extended in several directions. It is possible to have several types of entrepreneurs with different preferences and leverage, while maintaining analytical tractability. Such specification would allow the average level of risk-aversion to be time-varying, since positive shocks would redistribute resources towards agents with higher leverage and lower risk-aversion. In this case, a sequence of good shocks would decrease average risk-aversion and increase leverage, which might make economy more fragile to negative shocks.

Our framework also allows for contracts with optimal risk-sharing of aggregate risk between lenders and borrowers. In the current framework returns to lenders are predetermined, and entrepreneurs effectively carry all aggregate risk, so it would be interesting to investigate, whether the amplification of monetary and technology shocks is robust to the trade of state-contingent claims on the aggregate state of the world. From Dmitriev and Hoddenbagh (2013) and Carlstrom, Fuerst and Paustian (2013) and Carlstrom, Fuerst, Ortiz and Paustian (2014) we know that the financial accelerator is not robust to the presence of state-contingent contracts for risk-neutral entrepreneurs. Dmitriev and Hoddenbagh (2014) demonstrate that amplification is not robust to state-contingent contracts in costly enforcement environment, developed by Kiyotaki and Moore (1997) and extended by Iacoviello (2005) to risk-averse agents environment. The robustness of the accelerator to state-contingent contracts in costly state verification framework with risk-averse agents remains an important question for future research.
References


7 Appendix

7.1 Proof of Proposition 1

The proof follows Tamayo (2014). First, note that when the report is not verified ($\omega \not\in \Omega^V$) the repayment function must only depend on the report $\tilde{\omega}$, i.e. we have $R(\tilde{\omega})$. Therefore, the entrepreneur will choose $\omega^* = \arg\min_{\omega} R(\tilde{\omega})$ so the contract may as well set $R(\tilde{\omega}) = \tilde{R}$. Second, under the optimal contract, in the verification region $R(\omega) \leq \tilde{R}$ because otherwise the contract would not be incentive compatible. Specifically, the entrepreneur would prefer to misreport $\omega \not\in \Omega^V$ and pay $\tilde{R}$. Finally, it can also be shown that $\Omega^V$ must be a lower interval (for the proof see Lemma 3 in Tamayo (2014)). These findings can be summarized by saying that the optimal repayment function follows:

$$R(\omega) = \begin{cases} R(\omega) \leq \tilde{R}, & \text{if } \omega \leq \tilde{\omega} \\ \tilde{R}, & \text{if } \omega > \tilde{\omega} \end{cases}$$ (68)

Now let us rewrite the contracting problem using the above results as

$$\max \int_0^{\tilde{\omega}} \left( \kappa \frac{R_k}{\tilde{R}} \right)^{1-\rho} \left[ \omega - R(\omega) \right]^{1-\rho} d\Phi(\omega) + \int_{\tilde{\omega}}^{\infty} \left( \kappa \frac{R_k}{\tilde{R}} \right)^{1-\rho} \left[ \omega - \tilde{R} \right]^{1-\rho} d\Phi(\omega)$$ (69)

s.t. $\kappa \frac{R_k}{\tilde{R}} \left( \int_0^{\tilde{\omega}} R(\omega) d\Phi(\omega) + R[1 - \Phi(\tilde{\omega})] - \mu \Phi(\tilde{\omega}) \right) \geq (\kappa - 1)$ (70)

$\tilde{R} \leq \tilde{\omega}$ (71)

$R(\omega) \leq \omega \ \forall \ \omega \leq \tilde{\omega}$ (72)

$R(\omega) \geq 0 \ \forall \ \omega \leq \tilde{\omega}$ (73)

where we have plugged in the constraint (3), used the definition of leverage $\kappa = \frac{QK}{\bar{N}}$ and rescaled the objective function and constraints by the exogenous parameters $\bar{N}$ and $R$. Assign the multipliers $\lambda, \xi, \gamma_1(\omega)$ and $\gamma_2(\omega)$ to the constraints. The Lagrangian reads:
The first order necessary conditions with respect to $R(\omega), \bar{R}, \bar{\omega}$ after appropriate rescaling of the multipliers can be written as\textsuperscript{13}:

\begin{align*}
- \gamma_1(\omega)\phi(\omega) - \left(\frac{\kappa R^K}{R}\right)^{1-\rho} \{[\omega - R(\omega)]^{-\rho}\phi(\omega) + \lambda \left(\frac{\kappa R^K}{R}\right) \phi(\omega) + \gamma_2(\omega)\phi(\omega) = 0 \text{ for every } \omega \leq \bar{\omega} & \quad \text{(77)} \\
- \xi - \left(\frac{\kappa R^K}{R}\right)^{1-\rho} \int_{\bar{\omega}}^{\infty} [\omega - \bar{R}]^{-\rho}d\Phi(\omega) + \lambda \left(\frac{\kappa R^K}{R}\right) [1 - \Phi(\bar{\omega})] = 0 & \quad \text{(78)} \\
- \frac{\xi}{\phi(\bar{\omega})} - \left(\frac{\kappa R^K}{R}\right)^{1-\rho} [\bar{\omega} - R(\bar{\omega})]^{1-\rho} + \left(\frac{\kappa R^K}{R}\right)^{1-\rho} [\bar{\omega} - \bar{R}]^{1-\rho} - \lambda \left(\frac{\kappa R^K}{R}\right) [R(\bar{\omega}) - \bar{R} - \mu] = 0 & \quad \text{(79)}
\end{align*}

and the complementary slackness conditions:

\begin{align*}
0 &= \lambda \left\{ \frac{R^K}{R} \left( \int_{0}^{\bar{\omega}} R(\omega)d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu \Phi(\bar{\omega}) \right) \right\} - (\kappa - 1) \quad \text{(80)} \\
0 &= \xi [\bar{\omega} - \bar{R}] \quad \text{(81)} \\
0 &= \gamma_1(\omega)[\omega - R(\omega)] \quad \text{(82)} \\
0 &= \gamma_2(\omega)R(\omega) \quad \text{(83)}
\end{align*}

Suppose that $\gamma_1(\omega) > 0$ for all $\omega < \bar{\omega}$. Then it must be that $\gamma_2(\omega) = 0$, from the complementary slackness conditions. Then equation (75) would imply that $\lambda > \left(\frac{\kappa R^K}{R}\right)^{-\rho} (0)^{-\rho}$ which is not possible. Hence it must be true that $\gamma_1(\omega) = 0$ for all $\omega \leq \bar{\omega}$ and a standard debt contract is not optimal. We know from (75) that $\gamma_1(\omega) = 0 \iff (\omega - R(\omega))^{-\rho} \geq \lambda$. Now there are two possible cases. Suppose $\gamma_2(\omega) = 0$ for all $\omega \leq \bar{\omega}$. Then the contract specifies that

\textsuperscript{13}We do not need the first-order condition with respect to $\kappa$ to prove the proposition.
$R(\omega) = \omega - \lambda^{-1/\rho} \left( \frac{R}{R^k_{\sigma, t}} \right)$. By complementary slackness it should be the case that $R(\omega) > 0$ for all $\omega$, which is not possible because if $\omega = 0$, $R(\omega) > 0$ would not be feasible. Then it must be the case that $\gamma_2(\omega) > 0$ for some $\omega$ which implies $R(\omega) = 0$ and $\omega \leq \lambda^{-1/\rho} \left( \frac{R}{R^k_{\sigma, t}} \right)$ for the same $\omega$. Hence there is a lower interval where $R(\omega) = 0$. Call the upper bound of this interval $\omega \equiv \lambda^{-1/\rho} \left( \frac{R}{R^k_{\sigma, t}} \right)$. Therefore $R(\omega) = 0$ if $\omega \leq \omega$ and $R(\omega) = \omega - \omega$ if $\omega \leq \omega$. $lacksquare$

7.2 FOCs for the dynamic contract and proof of Proposition 2

The Lagrangian is

$$\mathcal{L} = \mathbb{E}_t \left\{ \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g(\bar{\omega}_{t+1}, \omega_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t})}{1-\rho} \Psi_{t+1} + \lambda_{t+1} \left( \kappa_t R_{t+1}^k h(\bar{\omega}_{t+1}, \omega_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) - (\kappa_t - 1) R_t \right) \right\}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \kappa_t} = \mathbb{E}_t \left\{ \left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{t+1} \Psi_{t+1} - \lambda_{t+1} R_t \right\} = 0 \quad (84)$$
$$\frac{\partial \mathcal{L}}{\partial \omega} = \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} + \lambda_{t+1} \kappa_t R_{t+1}^k h_{\omega,t+1} = 0 \quad (85)$$
$$\frac{\partial \mathcal{L}}{\partial \omega} = \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{\omega,t+1} \Psi_{t+1}}{1-\rho} + \lambda_{t+1} \kappa_t R_{t+1}^k h_{\omega,t+1} = 0 \quad (86)$$
$$\frac{\partial \mathcal{L}}{\partial R} = \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{\bar{R},t+1} \Psi_{t+1}}{1-\rho} + \lambda_{t+1} \kappa_t R_{t+1}^k h_{\bar{R},t+1} = 0 \quad (87)$$

Now we can express $\lambda_{t+1}$ from $\frac{\partial \mathcal{L}}{\partial \omega} = 0$

$$\lambda_{t+1} = \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R_{t+1}^k h_{\omega,t+1}} \quad (88)$$

Now we plug this condition into the three other equations and obtain

$$\frac{\partial \mathcal{L}}{\partial \kappa_t} = \mathbb{E}_t \left\{ \left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{t+1} \Psi_{t+1} + \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{\omega,t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R_{t+1}^k h_{\omega,t+1}} R_t \right\} = 0 \quad (89)$$
$$\frac{\partial \mathcal{L}}{\partial \omega} = \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} - \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{\omega,t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R_{t+1}^k h_{\omega,t+1}} \kappa_t R_{t+1}^k h_{\omega,t+1} = 0 \quad (90)$$
$$\frac{\partial \mathcal{L}}{\partial R} = \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{\bar{R},t+1} \Psi_{t+1}}{1-\rho} - \frac{\left( \kappa_t R_{t+1}^k \right)^{1-\rho} g_{\omega,t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R_{t+1}^k h_{\omega,t+1}} \kappa_t R_{t+1}^k h_{\bar{R},t+1} = 0 \quad (91)$$

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we can transform this system to

\[
\frac{\partial L}{\partial k_t} = \mathbb{E}_t \left\{ \left( R^k_{t+1} \right)^{1-\rho} \Psi_{t+1} \left( g_{t+1} + \frac{g_{\omega,t+1}}{1-\rho} R_t h_{\omega,t+1} \right) \right\} = 0 \tag{92}
\]

\[
\frac{\partial L}{\partial \omega} = g_{\omega,t+1} - g_{\bar{\omega},t+1} h_{\omega,t+1} = 0 \tag{93}
\]

\[
\frac{\partial L}{\partial \bar{R}} = g_{\bar{R},t+1} - g_{\bar{\omega},t+1} h_{\bar{R},t+1} = 0 \tag{94}
\]

Since in the equation (92) \(\Psi_{t+1}\) and \(\hat{R}_{k,t+1}\) enter as multiplicative terms and the term \(g_{t+1} + \frac{g_{\omega,t+1}}{1-\rho} R_t\) is equal to zero in the steady state, \(\Psi_{t+1}\) and \(\hat{R}_{k,t+1}\) have no effect in the first order approximation. Therefore, to find the approximate solution it is sufficient to consider the following system:

\[
\kappa_t R^k_{t+1} h_{\bar{\omega},t+1} (\bar{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) = (\kappa_t - 1) R_t \tag{95}
\]

\[
\mathbb{E}_t \left\{ g_{t+1} + \frac{g_{\omega,t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\omega,t+1}} R_t \right\} = 0 \tag{96}
\]

\[
\frac{g_{\omega,t+1}}{h_{\omega,t+1}} = \frac{g_{R,t+1}}{h_{\bar{R},t+1}} \tag{97}
\]

\[
\frac{g_{\omega,t+1}}{h_{\omega,t+1}} = \frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} \tag{98}
\]

We can substitute \(k_t\) and obtain

\[
\mathbb{E}_t \left[ \frac{g_{\bar{R}_{t+1}}}{(1-\rho)R_{k,t+1} h_{\omega,t+1}} \right] = \frac{1}{1 - \frac{R_{k,t+1}}{R_t} h_{\omega,t+1}} \tag{99}
\]

\[
\frac{g_{\omega,t+1}}{h_{\omega,t+1}} = \frac{g_{R,t+1}}{h_{\bar{R},t+1}} \tag{100}
\]

\[
\frac{g_{\omega,t+1}}{h_{\omega,t+1}} = \frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} \tag{101}
\]

Whenever the gradient of this system has full rank at the steady state, we will be able to find an approximate solution of \(\hat{\omega}_{t+1}, \hat{\bar{\omega}}_{t+1}, \hat{R}_{t+1}\) as functions of \(\mathbb{E}_t \bar{R}_{k,t+1} - \bar{R}_t, \bar{R}_{k,t+1} - \bar{R}_t\) and \(\bar{\sigma}_{\omega,t}\). Using this fact and log-linearizing equation (95) will give us
\[
\hat{k}_t = \nu_p(\mathbb{E}_t \hat{R}_{k,t+1} - \hat{R}_t) + \nu_\sigma \hat{\sigma}_{\omega,t}
\]
Figure 4: Overview of the Model

- Retailers
  - Wholesale Goods
  - Payments
  - Retail Goods
  - Capital, Entr. Labor
  - Retail Goods
  - Payments for Consumption

- Wholesalers
  - Payments
  - Labor
  - Wages
  - Capital Rent, Entrepreneurial Wage
  - Retail Goods

- Entrepreneurs
  - Capital, Entr. Labor
  - Payments for Capital

- Households
  - Dividends
  - Payment
  - Retail Goods
  - Payments for Consumption

- Financial Intermediaries
  - Deposits
  - Loans
  - Repayment

- Capital Producers
  - Capital
Figure 5: Impulse Response to Risk Shock

![Impulse Response Graphs](image-url)
Figure 6: Impulse Response to Wealth Shock
Figure 7: Impulse Response to Technology Shocks
Figure 8: Impulse Response to Monetary Shocks

- Consumption
- Leverage
- Investments
- Net Worth
- Output
- Borrowing
- Q

Nominal Interest Rate

Expected Excess Returns to Capital

< 0.28, \( \sigma = 0.5\)
< 0.085, \( \sigma = 0.5\)