Contract Complexity & Business Cycles

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July 2018

In the literature collateral constraints with non-contingent lending rates amplify business cycles. In particular, in recession resources redistribute from more effective borrowers to less effective lenders, which leads to deeper recession. We show that state-contingent interest rates prevent this redistribution. While moral hazard is a source for collateral constraints, contract complexity prevents the agents using optimal state contingent contract and interest rate reliefs. We develop a model that captures these phenomena by introducing a complexity penalty for adjustable rate contracts. We then show that complexity penalty is proportional to the cyclical difference between stochastic discount factors, and amplification monotonically increases with complexity penalty as agents move towards fixed rate contracts.

Keywords: Collateral constraints; financial accelerator; financial frictions; optimal contract.

JEL Classification Numbers: C68, E44, E61.

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1 Introduction

In a seminal paper, Kiyotaki and Moore (1997) show that collateral constraints amplify and propagate business cycle fluctuations. In their framework, negative shocks lead to a decline in the price of collateral, causing credit constraints to tighten. In this state of the world, entrepreneurs have lower revenues and less ability to borrow yet have to repay the same amount of debt due to a fixed rate contract agreed *ex ante*. Tighter credit constraints and the fixed rate contract exacerbate the initial effect of the negative shock on the economy, amplifying the impact of the shock and strengthening the resulting recession. This mechanism has been further developed by the costly state enforcement literature in Iacoviello (2005), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Quadrini (2014) and others.

In spite of the success of the collateral amplification mechanism, there is ongoing debate about its robustness to different types of lending contracts. For example, Krishnamurthy (2003), Nikolov(2014), and Cao and Nie (2017) find that amplification fails when agents can insure themselves against aggregate shocks in a stripped down two-state or three state version of Iacoviello (2005). They find that under state-contingent contracts, where the repayment rate depends on the aggregate state, amplification disappears. These conclusions are intuitive and consistent with Dmitriev and Hoddebagh (2017) and Carlstrom and Fuerst (2016), who find similar results in the costly state verification setup.

During a recession financially constrained entrepreneurs value internal funding more than during a boom not only because of consumption smoothing but also to avoid fire sales. Otherwise, borrowers write down the contracts where they have to repay lower interest rates during the recession to prevent fire-sales because of tighter collateral constraints. Thus, the literature has explained that both market incompleteness (regarding the number of state-contingent instruments) and collateral constraints are essential for amplification.

Economic agents often prefer fixed rate agreements due to the costly complexity of optimal state-contingent contracts. The goal of this paper is to study the costs of contract complexity on business cycles and collateral constraints. We develop a model that allows for the complexity penalty that increases with the conditional variance of the repayment interest rate. For example, the penalty under the fixed rate contract is zero. This simple setup allows capturing the fact that economic agents understand the fixed rate loans very well, and try to minimize the adjustable rate component. When adjustable rate loans provide some kind of insurance, the economic agents are willing to balance the benefits of insurance and a disutility from obfuscate agreements.

On the one hand, a high penalty parameter allows replicating the model with a fixed rate contract. On the other hand, when the penalty parameter approaches zero, the model generates the results that are indistinguishable from the complete markets of state-contingent securities with little amplification from collateral constraints. As this penalty goes up, the amplification from collateral constraints monotonically increases and eventually achieves the level under fixed rate contracts.

First, the main contribution of this paper is to develop the concept of complexity costs. It allows to study state-contingent contracts and the incompleteness wedge in a comprehensive way: one
parameter nests fixed rate and state-contingent contracts, as well as all intermediate cases. Second, we show that for all values of complexity costs, state-contingent contracts have a stabilizing effect on business cycles compare to fixed rate contracts. Third, we contribute to the understanding of the amplification mechanism by isolating incompleteness wedge from other wedges.

2 Credit Frictions in General Equilibrium

We now embed the three loan contracts in a dynamic New Keynesian model. The model consists of patient households who provide savings (which captures the assumption that households have lower discount rates than firms), entrepreneurs who borrow money from households and invest in projects, retailers who are the source of nominal rigidities, and central banks. Households work for entrepreneurs, who produce intermediate goods that require both labor and housing in the production process. Retailers bundle together the intermediate goods into a final consumption good.

Entrepreneurs

Entrepreneurs produce intermediate goods using a Cobb-Douglas production function composed of technology $A$, labor $L$ and real estate $h$:

$$Y_t = Ah_t^v L_t^{1-v}. \quad (1)$$

As in Bernanke, Gertler and Gilchrist (1999), we assume that retailers buy intermediate goods at the wholesale price $P^w_t$ and bundle them together in constant elasticity of substitution fashion into a final consumption good with price index $P_t$. We follow Iacoviello (2005) and define $X_t \equiv P_t / P^w_t$ as the markup of final over intermediate goods. The entrepreneur’s expected lifetime utility is given by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \gamma^t \ln c_t \quad (2)$$

where $c_t$ is entrepreneurial consumption and $\gamma > \beta$ so that entrepreneurs borrow money from patient households because they have a higher discount factor. The budget constraint for entrepreneurs is as follows:

$$Y_t/X_t + b_t - \frac{\alpha}{2} (rr_t - E_{t-1}rr_t)^2 = c_t + q_t \triangle h_t + rr_t b_{t-1} + \omega_t L_t. \quad (3)$$

Repayment interest rate $R_t$ in period $t$ is allowed to be contingent on the aggregate state in period $t$. However, contingency of the repayment comes at a costs reflected by the term $\alpha (r_t - E_{t-1}r_t)^2$. When the repayment interest rate in period $t$ is contingent only on the state $t-1$, the quadratic costs become zero.

Following Kiyotaki and Moore (1997) and Iacoviello (2005), we assume that the borrower can repudiate his debt in the by the end of the period when he borrowed the money. In this case lenders
can repossess the borrower’s housing assets by paying a proportional transaction costs $1 - m$ proportion of the assets. We assume that they will be able to sell the assets only in the next period. The borrowing constraint can be written as:

$$b_t \leq mh_t E_t(q_{t+1} \Lambda_{t,t+1}). \quad (4)$$

Finally, while the borrowers offer the lenders the amount they would like to borrow $b_t$ and the repayment schedule $rr_{t+1}$, lenders should have a participation constraint:

$$1 = E_t(rr_{t+1} \Lambda_{t,t+1}) \quad (5)$$

The participation constraint simply reflects that the lenders have an outside option of consuming one unit of output instead of lending it out to the borrowers. Now we are ready to formulate the optimization problem.

Entrepreneurs maximize their expected utility (2) subject to the budget constraint (3), borrowing constraint (4) and lenders’ participation constraint (5).

The corresponding Lagrangian is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \gamma' \left( \ln Y_t / X_t + b_t - \frac{\alpha}{2} (rr_t - E_{t-1}rr_t)^2 b_{t-1} - q_t \triangle h_t - rr_t b_{t-1} - w'_t L_t \right) + \lambda_t (mh_t E_t(q_{t+1} \Lambda_{t,t+1}) - b_t) + \zeta_t E_t(rr_{t+1} \Lambda_{t,t+1} - 1) \right\} \quad (6)$$

The first order conditions to this optimization problem with respect to debt, housing, labor and interest rate are given by:

$$\frac{1}{c_t} = \gamma \mathbb{E}_t \left\{ \frac{rr_{t+1} - \frac{\alpha}{2} (rr_{t+1} - E_t rr_{t+1})^2}{c_{t+1}} \right\} + \lambda_t \quad (7)$$

$$\frac{q_t}{c_t} = \mathbb{E}_t \left\{ \frac{\gamma}{c_{t+1}} \left[ v \frac{Y_{t+1}}{X_{t+1} h_t} + q_{t+1} \right] + m \lambda_t q_{t+1} \Lambda_{t,t+1} \right\} \quad (8)$$

$$w'_t = (1 - v) \frac{Y_t}{X_t L_t} \quad (9)$$

$$\zeta_t \Lambda_{t,t+1} = \gamma \frac{1}{c_{t+1}} b_t (1 + \alpha (rr_{t+1} - E_t rr_{t+1})) \quad (10)$$

Under the first order condition with respect to $b_t$, marginal benefits of borrowing should be equal to marginal costs. While marginal benefits include the marginal utility of consumption, marginal costs consists of the discounted lost future marginal utility of consumption multiplied by the state contingent repayment rate net of complexity costs and the shadow value of the borrowing constraint $\lambda_t$. Here the shadow value of borrowing constraints and penalty of complexity introduce an intertemporal wedge in addition to the standard components of an Euler equation. While the complexity costs are of the second order magnitude so that they drop from the log-linearization.
The Lagrangian multiplier, on the other hand, introduces a steady state and first order magnitude distortion.

For the first order condition with respect to housing to be satisfied, the marginal benefits of purchasing housing should be equal to marginal costs. Marginal benefits include the marginal product of consumption, the opportunity to sell the housing unit at the price $q_{t+1}$, and the opportunity to relax the borrowing constraint, where the last component of relaxing borrowing constraint introduces a housing allocation wedge. The costs have marginal utility of consumption multiplied by the housing price $q_t$. Time-varying markup causes another source of housing allocation wedge as entrepreneurs get only a fraction of marginal product of housing in production.

Labor demand is standard and implies that the wage should equal the marginal product of labor corrected by the markup from monopolistic competition. Time-varying markup $X_t$ here introduces a labor-consumption allocation wedge.

The last first order condition (10) with respect to $r_{t+1}$ define the optimal state contingent repayment rate. The optimal interest rate depends on marginal utility of consumption between the lender and the borrower. Here the complexity costs $\alpha(r_{t+1} - E_t r_{t+1})$ introduce the incompleteness wedge. One of the main goal of this paper is to investigate the role of this wedge for different degrees of $\alpha$.

**Proposition 1** Under the optimal contract, the surprise to the repayment rate is proportional to the difference in innovations of stochastic discount factors between lenders and borrowers. Equivalently, resources move towards the agents with the largest stochastic discount factors. Mathematically,

$$\hat{\Lambda}_{t,t+1} - E_t \hat{\Lambda}_{t,t+1} = \hat{U}_{c,t+1} - E_t \hat{U}_{c,t+1} + \alpha r_{t+1} - E_t \hat{r}_{t+1}.$$  \hspace{1cm} (11)

**Proof** Log-linearize (10) around the deterministic steady state and obtain $\hat{\Lambda}_{t,t+1}$. Then take the expectation of time $t$ from this expression to find $E_t \hat{\Lambda}_{t,t+1}$. The difference between $\hat{\Lambda}_{t,t+1}$ and $E_t \hat{\Lambda}_{t,t+1}$ should give (11).

This proposition reflects that under optimal contracts with complexity costs resources move towards the agents with the largest discount factor. In other words, to maximize welfare resources should move to the agents who needs them most subject to the complexity costs. Correspondingly, in the absence of contract complexity costs, discount factors should have perfect comovement or formally:

**Corollary 2** For $\alpha = 0$ entrepreneurs have with a full set of state-contingent contracts and a perfect comovement of marginal utility of consumption between households and entrepreneurs.

$$\hat{\Lambda}_{t,t+1} - E_t \hat{\Lambda}_{t,t+1} = \hat{U}_{c,t+1} - E_t \hat{U}_{c,t+1}.$$  \hspace{1cm} (12)

In the absence of complexity costs financial constraints can cause stochastic discount factor differ for agents only in the steady state, while across the cycle discount factors perfectly comove. Ef-
fectively, it means that if collateral constraints for borrowers get tighter, repayment rate should go down. Consequently, borrowers get a relief. Otherwise, discount factor for borrowers should go up stronger than for lenders.

Fixed rate in this case is simply the case with complexity costs going to infinity. As a result, the link between stochastic discount factors disappears. Formally,

**Corollary 3** As \( \alpha \to \infty \), the \( \rho_{t+1} \to E_t \rho_{t+1} \) so that the optimal contract has a predetermined rate of repayment.

In this case we replicate the standard results in the literature by Iacoviello and others. Under fixed rates we obtain the standard amplification: negative shocks causes a drop of housing price and tightening of the collateral constraint. In this case the borrower has to repay the same amount of money regardless of the shock, which forces him to cut on his consumption and housing, which further leads to a deterioration of housing prices and tightening of the constraint. Naturally, entrepreneur with the highest efficiency loses access to resources in recession. We observe the redistribution of goods from more efficient agents to less efficient agents as a consequence of a negative shock. It can only happen when stochastic discount factor relatively increases stronger for the most efficient agent in recession. Therefore, the combination of fixed rate contracts and collateral constraints amplifies business cycles. In the absence of collateral constraint entrepreneur would borrow more in recession, under the flexible rates he would get a relief in terms of lower interest rate.

**Patient Households**

Patient households maximize their lifetime expected utility

\[
\mathbb{E}_t \sum_{i=0}^{\infty} \left\{ \beta' \left( \ln c_i' + j \ln h_i' - (L_i')^\eta/\eta + \chi \ln(M_i/P_i) \right) \right\}
\]

where \( \beta \) is the household discount factor, \( c_i' \) is household consumption, \( h_i' \) is housing, \( L_i' \) denotes hours worked and \( M_i'/P_i \) denotes real money balances. The household budget constraint is as follows:

\[
c_i' + q_i \Delta h_i' + R_{t-1} b_{t-1} / \pi_t = b_i' + w_i' L_i' + F_i + T_i' - \Delta M_i'/P_i
\]

where \( \Delta \) is the first difference operator, \( q_i = Q_i/P_i \) denotes the real price of housing, \( b_i' \equiv B_i'/P_i \) is the real amount of borrowing, \( R_{t-1} \) is the nominal interest paid on loans between \( t-1 \) and \( t \), \( \pi_t \equiv P_t/P_{t-1} \) is the gross inflation rate, \( w_i' \equiv W_i'/P_i \) is the household real wage, \( F_i \) are lump-sum profits received from the retailer, and \( (T_i' - \Delta M_i'/P_i) \) are central bank transfers to household resulting from money printing. Households maximize their utility (13) subject to the budget constraint (14), choosing
consumption, loans, hours worked and housing, leading to the following first order conditions:

\[
\frac{1}{c_t} = \beta R_{t+1} E_t \left\{ \frac{1}{c_{t+1} \pi_{t+1}} \right\},
\]

\[w_t' = (L_t')^{\gamma-1} c_t',\]

\[
\frac{q_t}{c_t} = \frac{j}{h_t} + \beta E_t \left\{ \frac{q_{t+1}}{c_{t+1}} \right\}.
\]

These three equations yield the household consumption Euler condition, labor supply and housing demand.

**Retailers**

Sticky prices are introduced via the inclusion of a retail sector. Each retailer \(z\) purchases intermediate goods \(Y_t\) from entrepreneurs at the wholesale price \(P_t^w\) and then costlessly differentiates these goods into \(Y_t(z)\) and sell \(Y_t(z)\) at the price of \(P_t(z)\). Final goods are then aggregated in CES fashion according to the function \(Y_t^f = \left( \int_0^1 Y_t(z)^{\frac{1}{\epsilon}} \, dz \right)^{\frac{1}{\epsilon}}\) where \(\epsilon > 1\). The price index will be \(P_t = \left( P_t^{1-\epsilon}(z)dz \right)^{\frac{1}{1-\epsilon}}\).

The optimal reset price \(P_t^*\) is given by the solution to the following equation:

\[
\sum_{k=0}^\infty \theta^k E_t \left\{ \Lambda_t \left[ \frac{P_t^*(z)}{P_{t+k}^*} - \frac{X}{X_{t+k}} \right] Y_{t+k}^*(z) \right\} = 0
\]

where \(X_t\) is the markup, and \(X = (\epsilon - 1)/\epsilon\) is the steady state markup. The aggregate price level is defined by

\[
P_t = \left[ \theta P_{t-1}^{1-\epsilon} + (1 - \theta)(P_{t-1}^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}
\]

where \(1 - \theta\) is the probability that firms are able to reset their prices in a given period.

**Monetary Policy**

The central bank conducts monetary policy through a Taylor Rule of the following type

\[
\log(R_t) - \log(R) = \rho^R \left( \log(R_{t-1}) - \log(R) \right) + \rho^\pi \pi_t + \rho^Y \left( \log(Y_t) - \log(Y_{t-1}) \right) + \epsilon_t^R.
\]

where \(R\) is the steady state nominal interest rate, \(\rho^R\) and \(\rho^\pi\) determine the relative importance of the past interest rate and past inflation in the central bank’s interest rate rule. Shocks to the nominal interest rate are given by \(\epsilon^R\).
**Real Interest Rate**

The Euler equation for patient households in real terms is

\[
\frac{1}{c_i'} = \beta \mathbb{E}_t \left\{ \frac{r_{t+1}'}{c_{t+1}'} \right\}. \quad \text{(21)}
\]

Differently from the standard Euler equation, patient households agree to receive potentially non-predetermined interest rate \( r_{t+1}' \) in period \( t \). Clearly, with contract complexity \( \alpha \to \infty \) we move back to the standard Euler equation.

**Market Clearing**

Finally, we have market clearing in the goods, labor, housing and loan markets:

\[
Y_t = c_t + c_t', \quad \text{(22)}
\]

\[
L_t = L_t', \quad \text{(23)}
\]

\[
H_t = h_t + h_t', \quad \text{(24)}
\]

\[
b_t + b_t' = 0 \quad \text{(25)}
\]

**Shocks**

The shocks in the model follow a standard AR(1) process. The AR(1) process for technology is given by

\[
\log(A_t) = \rho \log(A_{t-1}) + \varepsilon_t^A, \quad \text{(26)}
\]

where \( \varepsilon_t^A \) denotes an exogenous shock to technology. Nominal interest rate shocks are defined by the Taylor Rule in (20).

**Equilibrium**

The model has 15 endogenous variables and 15 equations. The endogenous variables are: \( c, c', L, L', Y, Y', b, b', w', R, P, P^*, X, \lambda \), and \( q \). The equations defining these endogenous variables are (15), (16), (17), (1), (7), (8), (9), (18), (19), (20), (21), (22), (23), (24) and (25), and the equation defining the real interest rate is given by . The exogenous process for technology follows (26).

3 **Simulations**

Our baseline calibration largely follows Iacoviello (2005) and can be found in Table 1. We set the discount factor for households \( \beta = 0.99 \), the risk aversion parameter \( \sigma = 1 \) so that utility is logarithmic in consumption and the elasticity of labor \( \eta = 1.01 \). Housing enters the utility function with weight \( j = 0.1 \). Entrepreneurs have a discount factor \( \gamma = 0.98 \). Again, note that \( \beta > \gamma \) to ensure a binding collateral constraint. The share of housing in the Cobb-Douglas production function is
\( \nu = 0.03 \), while the loan-to-value ratio for entrepreneurs is 0.89.

For price-setting, we assume the Calvo parameter \( \theta = 0.75 \), so that only 25% of firms can reset their prices in each period, meaning the average length of time between price adjustments is four quarters. As our baseline, we follow the Iacoviello (2005) monetary policy rule and set the autoregressive parameter on the nominal interest rate to \( \rho^R = 0.73 \) and the parameter on current inflation to \( \rho^\pi = 0.27 \). We set the persistence of shocks to technology \( \rho^A = 0.95 \). In our quantitative analysis we compare two allocations: the competitive equilibrium under the debt contract with a predetermined lending rate and the competitive equilibrium under the state contingent lending rate.

Figure 1 shows impulse responses for a one percent technology shock. Under the fixed rate contract (labeled "Predetermined Real Rate" in Figure 1) real returns do not react to the shock. Borrowers can pledge more collateral during a boom as their collateral is now more valuable. The wealth of borrowers also increases substantially, and being impatient they expend as many resources as possible to buy more housing and consume while times are good. Therefore, consumption and investment climb dramatically under the fixed rate contract. On the other hand, under the optimal state-contingent contract (labeled “Optimal Indexation” in Figure 1) borrowers are only slightly better off, as higher returns from production are offset by the higher interest rate they must pay to lenders. This redistribution of resources dampens the response of output, consumption and investment to the initial shock. Under the optimal state-contingent contract a small welfare redistribution leads to lower inflation, lower output, less lending and lower entrepreneurial housing than the response of these variables under the fixed rate contract.

Figure 2 plots impulse responses for a 25 basis point (annualized) decline in the nominal interest rate. The positive monetary shock increases the inflation rate, along with output, lending and the housing price. The response of these variables is much stronger under the fixed rate contract than the optimal state-contingent contract. As in the case of a technology shock, the entrepreneurial share of the housing market does not react to a monetary shock. Again, the key difference between the two lending contracts is illustrated through the real returns on the loan. In the case of the fixed rate contract, real returns do not react to the shock on impact, so that housing prices and quantities increase substantially. As households feel much wealthier, they borrow much more to finance further consumption and investment, leading to a large positive impact on output. This effect is significantly dampened under the optimal state-contingent contract, where real returns increase on impact, forcing borrowers to repay higher amounts and preventing a large increase in house prices and borrowing.

Figure ?? compares the output response of the fixed rate contract and the optimal state-contingent contract against a frictionless benchmark. As our frictionless benchmark, we take the share of entrepreneurs in the economy to zero and set the collateral constraint coefficient \( m \) to infinity, so that the model converges to the standard New Keynesian framework with only patient households. Relative to the frictionless benchmark, the fixed rate contract generates much larger amplification for both technology and monetary shocks. In contrast, the optimal state-contingent contract generates a similar amount of amplification to the frictionless benchmark, and amplification is even smaller in
some periods following the shock. The stabilizing influence of the optimal state-contingent lending contract completely eliminates the collateral amplification mechanism.

4 Conclusion

This paper contributes to the literature on financial frictions in macroeconomics by introducing state-contingent lending contracts to the collateral constraint framework of Kiyotaki and Moore (1997). In their framework, a negative shock leads to a feedback loop of deteriorating collateral values and tighter credit constraints. We show that two key assumptions drive this collateral amplification mechanism. First, the interest rate on loans is non-contingent and does not vary with the aggregate state of the economy. Second, borrowers do not have access to secondary financial markets where they can trade aggregate risk. We relax both assumptions and find no amplification relative to the basic New Keynesian model, even in the presence of collateral constraints. Our results confirm Krishnamurthy’s (2003) findings in a fully dynamic, quantitative business cycle framework, and also match the ability of state-contingent lending contracts to remove the amplifying property of financial accelerator mechanisms in models of adverse selection (House (2006)) and costly state verification (Dmitriev and Hoddenbagh (2013)).
References


### Table 1: Calibration

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<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
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<td>Patient households</td>
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<td>Entrepreneurs</td>
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<td>Housing Share</td>
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<td>Loan-to-value entrepreneur</td>
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<td>Monetary policy Inflation Gap</td>
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Figure 1: Technology Shock

Figure 2: Monetary Shock
Figure 3: Response of Output to Shocks on Impact for Varying Degrees of Complexity Penalty